Towards Spatio-Temporal Semantics

- Characterize motion qualitatively
- Handle time by snapshots of the world (states) (as opposed to domains as “object-histories”)
- Instance of CSP(Γ): primitive positive formulas without free variables
- Instance of SeqCSP(Γ): S = (V, (Q₁, . . . , Q₄)) where the Qᵢ are instance of CSP(Γ)

Definition of Transitions

- T₂-solution of SeqCSP(Γ):
  \[ \alpha^t(v_i) \prec \alpha^t(v_j) \implies \alpha^{t+1}(v_i) \leq \alpha^{t+1}(v_j). \]
  \[ Q^1 \frac{y}{x} \frac{z}{y} \]
- T₁-solution of SeqCSP(Γ): T₂-solution such that
  \[ \alpha^t(v_i) \neq \alpha^t(v_j) \land \alpha^t(v_i) = \alpha^t(v_j) \implies \neg (\alpha^{t+1}(v_i) = \alpha^{t+1}(v_j) \land \alpha^{t+1}(v_k) \neq \alpha^{t+1}(v_l)) \]
- It does not allow point-to-interval and interval-to-point transitions at the same time.

The Complexity of Continuity

- Constraint languages on infinite domains \( D \)
  \( \Gamma = \langle D, R^D_1, \ldots, R^D_n \rangle \)
- Knowledge as primitive positive formulas:
  \( \varphi = \exists x_1 \ldots x_m : \bigwedge_j R_j x_{i_1} \ldots x_{i_j} \)
- Complexity of the satisfiability problem depends on \( \Gamma; P, NP, \ldots, \) undecidable

Example: Point Algebra
- base algebra \((Q, \prec)\)
- full algebra \((Q, \prec, \leq, \neq)\)
Interpret \( \prec \) spatially (in front/behind) and in multiple dimensions

PA is central
Interval Algebra, Cardinal Directions, Block Algebra, Rectangle Algebra can be expressed by PA

Unrestricted relational Languages

- Qualitative descriptions should handle (immediate) continuity in motion: \( x \prec y \iff x > y \)
- Associated with neighborhood graphs [Freksa, 1991]
- Represent continuity as 2 · n-ary relations: \( T_2/T_1 \subseteq D_{2n}^n \)
- Neighborhood graphs provide merely binary projection of \( T_2/T_1 \)

Where to go from Here?

- Analyzing continuity in non point-based formalims (RCC, OCC, etc.)
- Investigating state space approach is close to AI planning, e.g., PDDL
- Integrating continuity constraints into SAT/CP

References

- M. Bodirsky, J. Kára, ‘The complexity of temporal constraint satisfaction problems’, JACM 57(2), 2010
- A. Galton, Qualitative Spatial Change, Oxford Press, 2000